UDC 532.7:534.1 DOI: https://doi.org/10.15407/geotm2023.165.036

MASS EXCHANGE IN TWO-LAYER MEDIUM MOVING THROUGH THE NARROW CYLINDRICAL CHANNEL

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Abstract. This work further develops previous studies devoted to numerical modeling of diffuse mass transfer in narrow pore channels. The problem of diffusion in a two-layer liquid moving through narrow cylindrical channel, into which a neutral component, which does not interact with heterogeneous inclusions, diffuses, is considered. The dispersion carrier fluid moves close to the wal, while a reologically complicated two-phase medium occupies most of the channel. During the flow of a fine-dispersed concentrated suspension in a quasi-balanced condition, the reological properties of the medium were accepted as parameters of some homogenous liquid, which can be deemed an adequate approximation. This model can be used for some situations of flow in the chanal of fluid bodies which are capillary-porous and broken, such as those that contain muddy or clay inclusions. Similar mathematical models can be applied to paste sliding flows because of poor capillary fluid fixation. In this paper, two cases are considered. In the first scenario, a portion of the channel midsection is exceeded by the diffusing component. In the second, this component in the same location exits the channel. The non-stable problem is numerically solved before the creation of the stationary state. The installation procedure was monitored up to the fifth decimal digit. The resulting solution determines the concentration fields of the diffuse component. It is demonstrated that distribution of the diffuse component concentration in the working area is influenced by the rate of the medium speed; diffuse flow through the wall, and effective diffusion coefficient. In this work, the case is considered when there is no interaction between the diffusing component and the dispersed medium. However, the interaction of these components of the medium in soils, biological systems, and natural layers containing organic inclusions is of great interest. Therefore, accounting of adsorption properties of the dispersed medium in relation to the elements involved in mass exchange can be in the focus of future study. Taking into account large-scale metabolic processes, such as those in blood in the veins, is crucial in many practically significant circumstances.

Keywords: capillary, liquid, dispersed medium, diffusion, mass exchange.

1. Introduction

Mass transfer is the basis of many technological processes in multiphase and capillary-porous systems, which can be both an independent process and an accompanying or preparatory one for bringing the system to a particular state. In the first case, an example can be a process such as extraction [1], when it is necessary to obtain a specific component; in the second - hydrogenation of coal [2] to prepare material for the following technological stages of its or entire coal seams [3] use to change, for example, their strength properties or reduce explosiveness.

There are many different techniques to describe the heat and mass transfer processes in such environments [4] depending on the size of the regions and the size of the components that make up the capillary porous media. In most cases, it is essential to consider the process in terms of a particular characteristic element of the pore system in order to describe it in detail. A narrow channel or a capillary is a typical example of such element, for which the mathematical statement is simplified and becomes convenient for numerical analysis. On the one hand, this channel can simulate lengthy fissures or cracks in natural rocks, which is a subject of great attention for today [5]. On the other hand, the capillary system should also be considered because it is the basis for nutrient medium moving in plants and blood moving in the organisms of animals and humans.

Currently, considerable attention is paid to the dynamics of blood circulation, which takes place in the complex branched system of vessels. For mathematical study of mass transfer at blood moving, a significant difficulty is to determine rheological properties of the moving liquid. As it is known [6, 7], blood consists of plasma (liquid) and hematocrit (blood corpuscles). The combination of these essential blood components brings great diversity to the dynamics of the moving medium. In the simplest case, the moving fluid is believed to be Newtonian with a particular value of dynamic viscosity [8]. When detailing the movement through different areas of the vessels (the diversity in sizes is large [9]), complications are introduced into the rheological properties. In order to explain how liquid moves through cylinder-shaped channels, non-Newtonian models of a step nature are used in [10–12]; viscoelastic features are also taken into account in [13]. The sizes of the shaped elements are of the order of 10 m, so their influence on the flow depends on the diameters of the vessels. The smaller is the diameter, the naturally more significant is the impact of the hematocrit on the movement. This explains why descriptions of rheological behaviour of blood are presented in such a wide range [6, 13, 14]. For example, erythrocyte flexibility, which enables them to move through channels even smaller in diameter than they are, is one of the most crucial characteristics of the blood corpuscle [6, 7]. Heterogeneous medium may be accepted as homogeneous in some circumstances, and, as a result, basic rheological models may be used.

Two-layer models, in which the flow of blood is assumed to be the flow of a Newtonian fluid but with varying values of dynamic viscosity in the layers [8, 10, 16–18], have become common in the literature. It is thought that the viscosity is equal to plasma viscosity at the near-surface zone, whose width is assumed to be lower than the diameter of the erythrocyte, and 3–4 times higher in the remaining portion. The region-based division and constant viscosity coefficients can make the study of mass transfer processes easier. The created mathematical models of flows in the cited literature can be used as an example for their application during technical or natural water flow, as well as some pastes saturated with a fine-disperse media in a quasi-equilibrium thermodynamic zone.

In this work, we consider diffusion of a passive component in a cylinder-shaped channel through which a two-layer liquid flows. To facilitate the diffusion process, we assume that the core zone of the flow is heterogeneous. Thus, mass transfer processes that can take place in these contexts are included in the flow models that are already being employed.

2. Methods

The equation of laminar motion in narrow cylindrical channel of constant diameter is well-known [19]

$$0 = -\frac{dp}{dz} + \mu_I \left(\frac{\partial^2 u_I}{\partial r^2} + \frac{\partial u_I}{r \partial r} \right), \tag{1}$$

where z – longitudinal coordinate, m; r – radius, m; p – pressure, Pa; u – speed, m/s; μ – coefficient of dynamic viscosity, Pa·s; the index I defines the current areas: 1 – peripheral, 2 – central. It follows from this equation that

$$u_1 = -\frac{dp}{4\mu_1 dz}\left(1 - n^2\right), \tag{2}$$

$$u_2 = -\frac{dp}{4\mu_1 dz}\left(1 - n_*^2 + \frac{\mu_1}{\mu_2}\left(n_*^2 - n^2\right)\right). \tag{3}$$

Here $n = r/Rc$, $n^* = r^*/Rc$, Rc is the capillary radius, m; r^* is the radius of the two-phase zone, m. The average speed, in this case, will be equal to

$$U_{SR} = -\frac{dp}{4\mu_1 dz}\left\{1 - n_*^2 - \frac{1}{2}\left(1 - n_*^4\right) + \left[\left(1 - n_*^2\right) + \frac{1}{2}\frac{\mu_1}{\mu_2} n_*^2\right] n_*^2\right\}. \tag{4}$$

The mass transfer equation can be written in the following form

$$\varepsilon\left(\frac{\partial c_I}{\partial t} + u_I \frac{\partial c_I}{\partial z}\right) = \left(\varepsilon d_I D\right)\left(\frac{\partial^2 c_I}{\partial r^2} + \frac{\partial c_I}{r\partial r} + \frac{\partial^2 c_I}{\partial z^2}\right), \tag{5}$$

where c_I is the concentration of the inert component; ε – porosity (in section 1 ε = 1); $d_I = D_I/D$; d_1 – relative diffusion coefficient in the liquid; d_2 – relative effective diffusion coefficient in section 2; D - scale coefficient of diffusion in the liquid, m^2/s; D_1 – diffusion coefficient in the carrier liquid, m^2/s. The effective diffusion coefficient D_2, according to [19], depends on the structure of the pore space (in this case, the value of this coefficient was considered isotropic and was set).

Two tasks were considered. In the first one, it was assumed that the flow moves with an average speed determined by equation (4), and some component diffuses into the channel at some distance z^* from the beginning, the concentration of which is constant on the surface of the vessel, that is, for $0 \le z < z^*$ $c_I = 0$, for $z \ge z^*$ $c_I = c^*$.

In the second task, it was also assumed that the flow moves with the average U_{SR} speed and the concentration in the initial section is constant, and for $z \ge z^*$ the component diffuses into the channel wall according to the dependence $-D_1\left(\partial c_1 / \partial r\right)_{r=Rc} = kc_1$. At the boundary of the layers, the condition $c_I = c_2$ and the condition of conservation of the mass of the diffusing component are fulfilled.

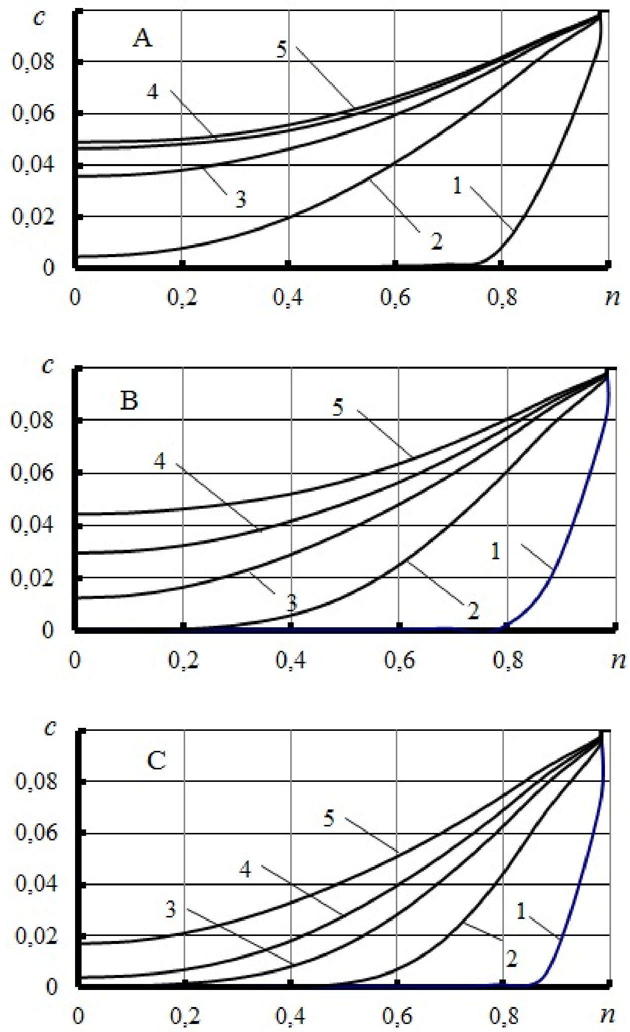
$$D_1 \frac{\partial c_1}{\partial r} = \left(\varepsilon D_2\right)\frac{\partial c_2}{\partial r}, \tag{6}$$

$$\frac{\partial c_2}{\partial r} = 0\,. \tag{7}$$

3. Results and discussion

Equations (1)–(5) with boundary conditions (6), (7) were solved numerically by analogy with [20, 21]. As a result, the steady-state concentration curves shown in figures 1– 4 for the tube with radius of *Rc* = 0.00025 m and length of *L* = 0.1 m ($\zeta = z/L$) were obtained. More exactly, figures 1 and 2 show the results of calculations for the first problem, and figures 3 and 4 show the results of the second problem.

Figure 1 shows concentration curves for the following velocities: *Usr* = 0.001 m/s (A); *Usr* = 0.002 m/s (B); *Usr* = 0.005 m/s (C) for the case ε = 0.6, d_1 =v1, d_2 = 0.5, $D = 10^{-9}$ m^2/s.

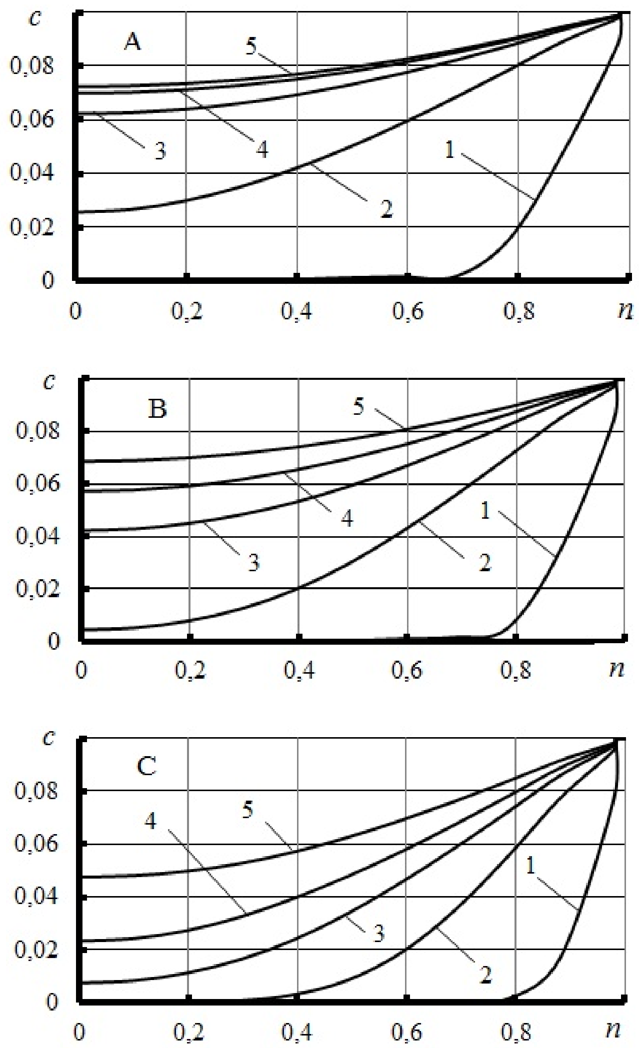


1 – ζ = 0.1; 2 – ζ = 0.2; 3 – ζ = 0.4; 4 – ζ = 0.6; 5 – ζ = 1

Figure 1 – Concentration of the component in the cross sections of the tube

Figure 1 demonstrates how the flow rate greatly influences the component filling the cross section in the tube. Of course, the diffusion coefficient is one of the crucial factors here. It is well known that the effective diffusion coefficient for capillary-porous bodies can be considerably less than the value in the medium that fills the body pores [4]. Although the dispersed phase is presumed to be flexible in this medium, small-scale pulsations are possible intensifying the diffusion process and making the effective coefficient to be significantly higher.

Figure 2 shows the same concentration curves, but for $d_2 = 1$.

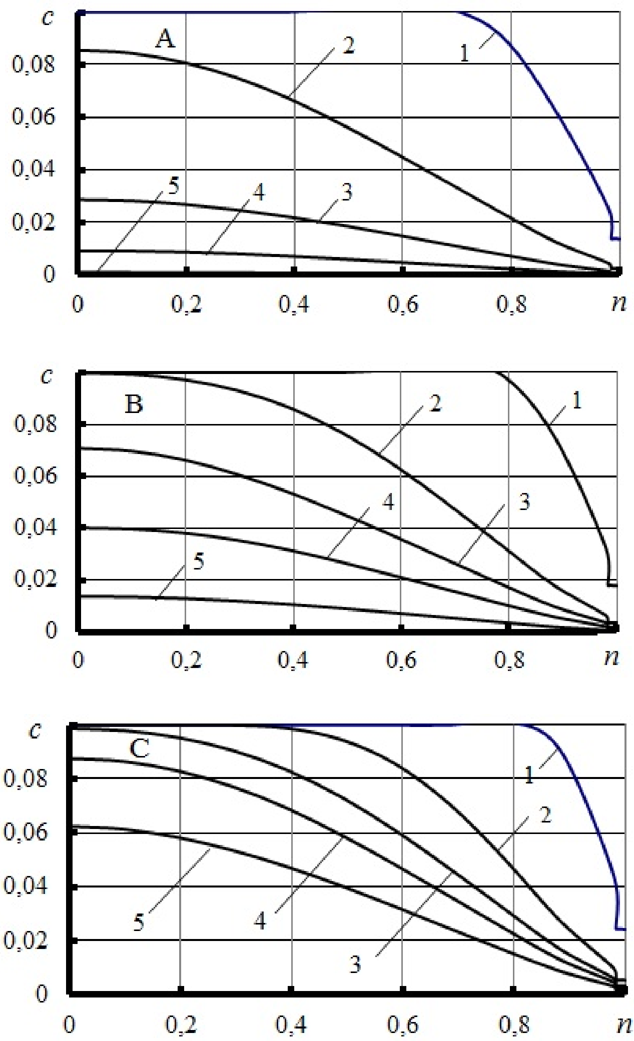


1 – ζ = 0.1; 2 – ζ = 0.2; 3 – ζ = 0.4; 4 – ζ = 0.6; 5 – ζ = 1

Figure 2 – Concentration of the component in the cross sections of the tube

Comparing the curves in figures 1 and 2, it is clear that an increase in the effective diffusion coefficient naturally leads to a faster filling of the cross section of the tube. Thus, when studying mass transfer in a medium with some inclusions, the important issue is to determine the effective diffusion coefficient. Further, the solution of the problem was also carried out for a porosity of 0.4 (assuming that the effective diffusion coefficient is constant). The findings demonstrated (though not in the figures) that the steady diffusion process is practically unaffected by the magnitude of the porosity. This is because, in equation (5), the porosity is reduced and only remains in the equality of diffusion flows (6), which results in a decrease in the flow rate of the diffusing component. However, for this issue, this decrease in flow rate is not crucial because the free volume in which this component diffuses also shrank.

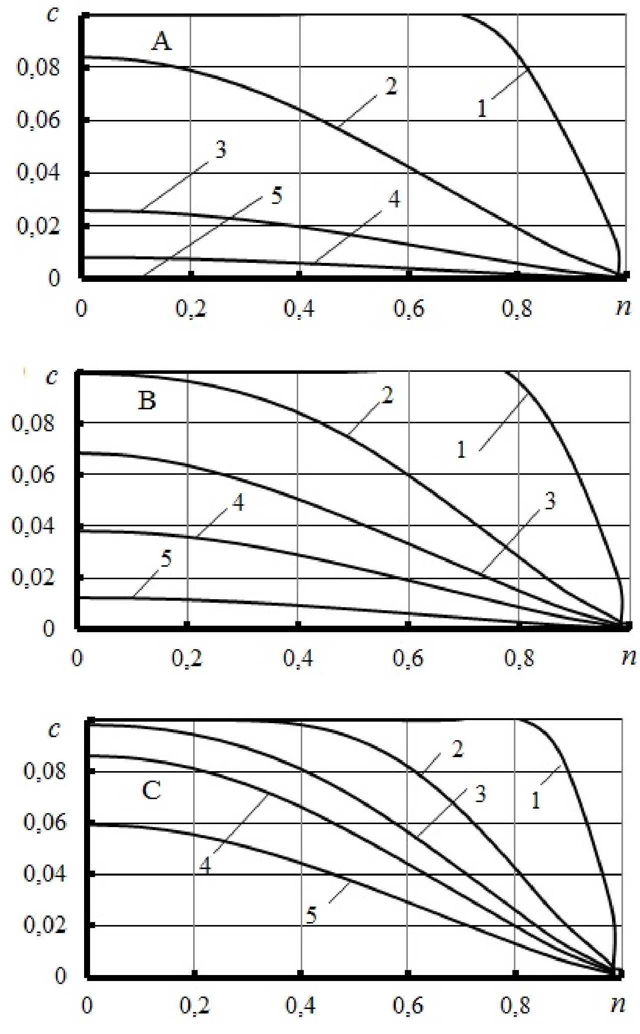
The next two figures 3 and 4 show the results of calculations of the second problem, i.e. the process of the exit of the component from the cylindrical channel. Here, the diffusion flow on the channel wall ($r = Rc$) is taken as $q = -D_1 \left.\frac{\partial c_1}{\partial r}\right|_{r=Rc} = kc_1$ where k is the coefficient, m/s. Figure 3 shows the results of calculations for the value of the coefficient k = 0.0001 m/s, figure 4 - for k = 0.001 m/s. Figure 3 shows the curves of changes in the concentration of the diffusing component in cross sections at different flow rates. At the end of the curves in the right part, small shelves are clearly visible, which indicate that the concentrations are practically uniform in the clean wall zone.



1 – ζ = 0.1; 2 – ζ = 0.2; 3 – ζ = 0.4; 4 – ζ = 0.6; 5 – ζ = 1

Figure 3 - Concentration of the component in the cross sections of the tube (k = 0.0001 m/s)

As it can be seen from figure 3, the magnitude of the velocity naturally has a significant effect on the concentration distribution. It can be seen from curves 5 that at a speed of 0.001 m/s the substance almost completely leaves the channel, while at a speed of 0.005 m/s a lot of it still remains in the porous area. The value of the coefficient *k* in this version of the calculations for *Usr* = 0.001 m/s turned out to be practically marginal, in the sense that the component practically in full volume leaves the calculated working area. Figure 4A shows similar curves for the case *k* = 0.001 m/s. It can be seen that the concentration curves are located somewhat lower than in figure 3A, that is, it is practically close to the variant with catalytic wall, when the concentration on the surface would be zero. The results of the calculations, shown in figures 3 and 4, indicate that the velocity of the medium plays an important role, the value of the diffusion flux through the wall naturally also determines the diffusion process in the regions. Finally, calculations of this problem with different porosity showed (not shown in the figures) a weak dependence on its value. The concentration values, for example, for ε = 0.4, turned out to be somewhat lower than in the given figures 3, and 4 for ε = 0.6.



1 - ζ = 0.1; 2 – ζ = 0.2; 3 – ζ = 0.4; 4 – ζ = 0.6; 5 – ζ = 1

Figure 4 - Concentration of the component in the cross-sections of the tube (*k* = 0.001 m/s)

4. Conclusions

The diffusion process in two-layer medium flowing through the narrow cylindrical channel was considered. The results of the numerical calculations showed that the flow rate and diffusion coefficients had an impact on how the diffusing component was dispersed over the working area.

It should be noted that when considering mass exchange processes in many practically significant situations, such as blood flow through vessels, it is also necessary to know and to take into account adsorption properties of the dispersed medium in relation to the components involved in the mass exchange. This might be a future research topic.

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МАСООБМІН У ДВОШАРОВОМУ СЕРЕДОВИЩІ, ЩО РУХАЄТЬСЯ У ВУЗЬКОМУ ЦИЛІНДРИЧНОМУ КАНАЛІ

Єлісєєв В.І., Луценко В.І., Совіт Ю.П., Геті К.В.

Анотація. Ця робота є подальшим розвитком попередніх досліджень, присвячених чисельному моделюванню дифузійного масообміну у вузьких порових каналах. Розглянуто дифузійне завдання у двошаровій рідині, що рухається у вузькому циліндричному каналі, в яку дифундує нейтральна компонента, що не взаємодіє з гетерогенними включеннями. Більшу частину каналу займає реологічно складне двофазне середовище, а безпосередньо біля стінки рухається дисперсійна несуча рідина. При цьому реологічні властивості середовища були прийняті як параметри деякої гомогенної рідини, що можна вважати прийнятним наближенням під час течії дрібнодисперсних концентрованих суспензій в квазірівноважному стані. Цю модель можна поширити на деякі випадки течій у каналах, капілярно-пористих і тріщинуватих тілах рідин, наприклад, з мулистими або глинистими включеннями. Подібні математичні моделі можуть бути використані під час течії паст з прослизанням, обумовленим слабким закріпленням капілярної рідини. У цій роботі розглянуто два випадки. У першому випадку дифундуюча компонента на деякій ділянці надходить у середину каналу. У другому - ця компонента на такій самій ділянці йде з каналу. Нестаціонарне завдання вирішено чисельно до встановлення стаціонарного стану. Процес встановлення простежувався до п'ятого знака після коми. В результаті рішення визначено поля концентрацій дифузного компонента. Показано вплив величин швидкості середовища, дифузійного потоку через стінку та ефективного коефіцієнта дифузії на розподіл концентрації дифузного компонента в робочому просторі. Тут розглянуто випадок, коли між дифундуючим компонентом і дисперсним середовищем немає взаємодії. Однак у грунтах, біологічних системах та у природних пластах з органічними включеннями, значний інтерес представляє взаємодія цих компонентів середовища. Тому подальшим напрямом досліджень може бути облік адсорбційних властивостей дисперсного середовища по відношенню до компонентів, що беруть участь у масообміні. Це важливо також у багатьох практично значущих випадках під час розгляду масообмінних процесів, наприклад, у крові в судинах.

Ключові слова: капіляр, рідина, дисперсне середовище, дифузія, масообмін.